⑧ Find F T for $g(t) = Sgn(t)$

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-j2\pi ft} \, dt$$

$$= \int_{-\infty}^{0} -1\, e^{-j2\pi ft} \, dt + \int_{0}^{\infty} 1\, e^{-j2\pi ft} \, dt$$

$$= -\frac{e^{-j2\pi ft}}{-j2\pi f}\Big|_{-\infty}^{0} + \frac{e^{-j2\pi ft}}{-j2\pi f}\Big|_{0}^{\infty}$$

$$= \frac{1}{j2\pi f}\left[e^{0} - e^{\infty}\right] - \frac{1}{j2\pi f}\left[e^{-\infty} - e^{0}\right]$$
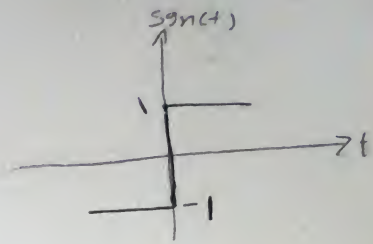
$$G(f) = \infty \quad \times$$

$$m(t) \xrightarrow{\text{relation}} Sgn(t)$$

$$F[Sgn(t)] \longrightarrow M(f)$$

$$Sgn(t) = \lim_{\alpha \to 0} m(t)$$

⇓

$$F[Sgn(t)] = \lim_{\alpha \to 0} M(f)$$

$$m(t) = e^{-\alpha t} u(t) - e^{\alpha t} u(-t)$$

⇒ Using Super Position

$$e^{-\alpha t} u(t) = \frac{1}{\textcircled{\alpha} + j2\pi f}$$

-١-
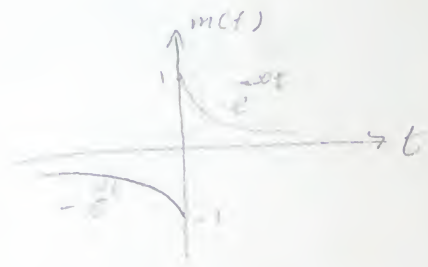
$$e^{\alpha t} \, u(-t) \longrightarrow \frac{1}{\alpha - j2\pi f}$$

$$M(f) = \frac{1}{\alpha + j2\pi f} - \frac{1}{\alpha - j2\pi f}$$

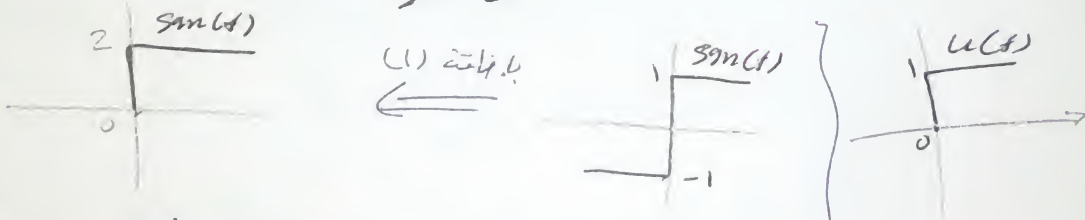$$M(f) = \frac{-j \, 4\pi f}{\alpha^2 + 4\pi^2 f^2}$$

$$F\left[sgn(t)\right] = \lim_{\alpha \to 0} M(f) = \frac{-j \, 4\pi f}{4\pi^2 f^2}$$
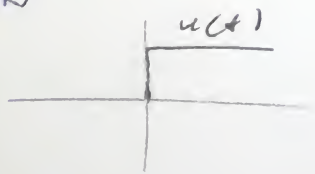
$$= \frac{-1}{\pi f} = \frac{1}{j\pi f}$$

$$\boxed{sgn(t) \rightleftharpoons \frac{1}{j\pi f}}$$

---

② find F.T for $g(t) = u(t)$

Soln

$$u(t) = \frac{1}{2}\left[sgn(t) + 1\right]$$

$$F\left[u(t)\right] = F\left[\frac{1}{2}\left(sgn(t) + 1\right)\right]$$

$$F[u(t)] = F\left[\frac{1}{2} \operatorname{sgn}(t) + \frac{1}{2}\right]$$

$$= \frac{1}{2} \frac{1}{J\pi f} + \frac{1}{2} \delta(f)$$

$$A \Longrightarrow A\,\delta(f)$$
$$\frac{1}{2} \Longrightarrow \frac{1}{2}\,\delta(f)$$
$$1 \Longrightarrow \delta(f)$$

③ Duality:-

If $g(t) \rightleftharpoons G(f)$

$$G(t) \rightleftharpoons g(-f)$$

Ex:- find FT of $g(t) = A\tau \operatorname{sinc}(t\tau)$

Soln

using duality

$$A\operatorname{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau\,(\operatorname{sinc}(f\tau))$$

$$A\tau\operatorname{sinc}(t\tau) \rightleftharpoons A\operatorname{rect}\left(\frac{-f}{\tau}\right)$$

$$A \, \text{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau \, \text{sinc}(f\tau)$$

$$A\tau \, \text{sinc}(t\tau) \rightleftharpoons A \, \text{rect}\left(\frac{-f}{\tau}\right)$$

center

$-f = 0$

قيمتهم ± 1 *

بتساوي بالعكس

الحصول على الـ center
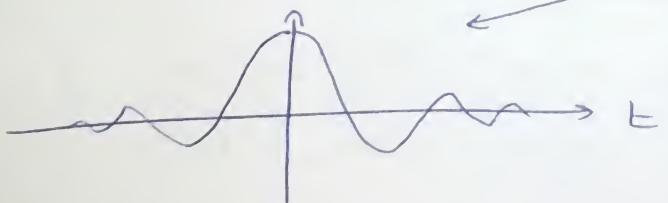
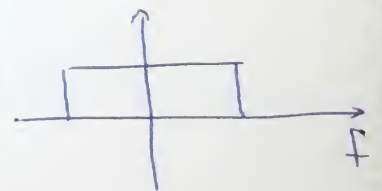$f = 0$
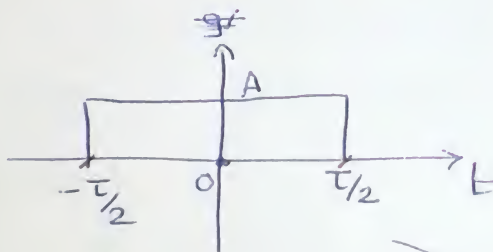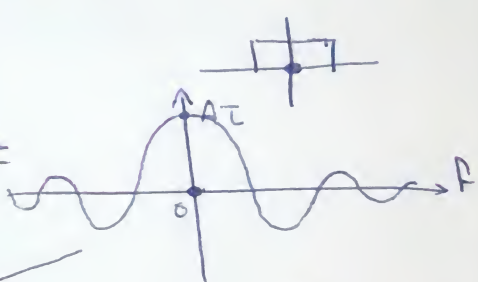
Limited in time $\longrightarrow$ unlimited in freq.

والعكس

(13) find F.T. for $g(t) = A \, \text{sinc}(2\omega t)$.

using duality

$$A \, \text{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau \, \text{sinc}(f\tau)$$

$$\longrightarrow A\tau \, \text{sinc}(t\tau) \rightleftharpoons A \, \text{rect}\left(\frac{f}{\tau}\right)$$

$\div \tau$

$$A \, \text{sinc}(2\omega t) \rightleftharpoons \frac{A}{2\omega} \, \text{rect}\left(\frac{f}{2\omega}\right)$$

$\tau = 2\omega$

Find F.T. of $g(t) = \text{sinc}(mt)$

using duality

$$A \, \text{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau \, \text{sinc}(f\tau)$$

$$A\tau \, \text{sinc}(t\tau) \rightleftharpoons A \, \text{rect}\left(\frac{f}{\tau}\right)$$

$$1 \cdot \text{sinc}(mt) \rightleftharpoons \frac{1}{m} \, \text{rect}\left(\frac{f}{m}\right)$$

$\tau = m$

④ Time Shift Property

$$\text{If} \quad g(t) \rightleftharpoons G(f)$$

$$g(t \underset{\ominus}{\oplus} t_0) \rightleftharpoons G(f) \cdot e^{\pm J2\pi f\, t_0}$$

نفس الاشارة

Find F.T. for $g(t) = A \, \text{rect}\left(\dfrac{t - t_0}{\tau}\right) \xrightarrow{\;=0\;} \begin{array}{l} t = t_0 \\ center \end{array}$
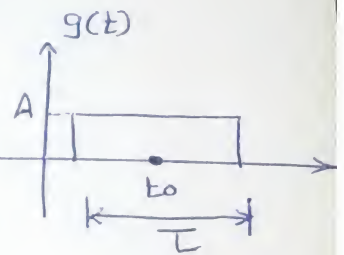
↓
Amp.   └→ الموقع

using time shift

$\therefore G(f) = A\tau \, \text{sinc}(f\tau) \cdot e^{-J2\pi f\, t_0}$

$\therefore \quad A \, \text{rect}\left(\dfrac{t}{\tau}\right) \rightleftharpoons A\tau \, \text{sinc}(f\tau)$

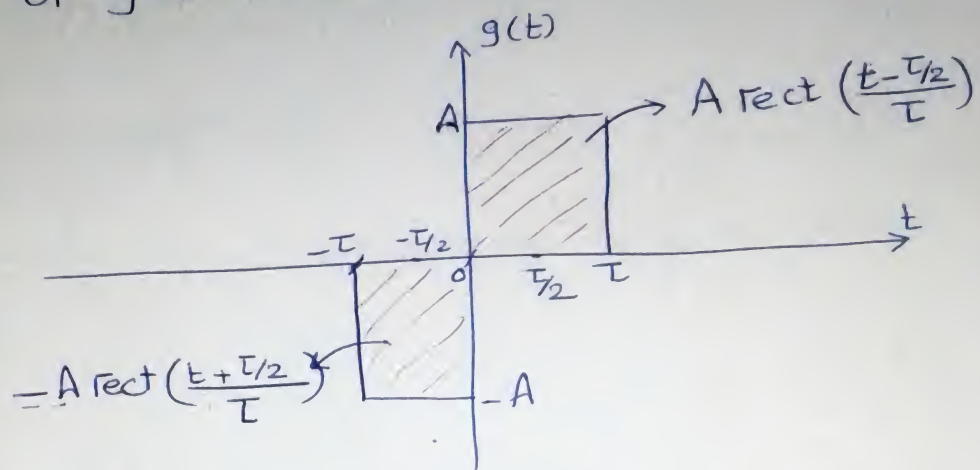$\underset{g(t)}{} \qquad\qquad \underset{G(f)}{}$

Find F.T. for $g(t) = A \, \text{rect}\left(\dfrac{t - \tau/2}{\tau}\right)$

using time shift

$G(f) = A\tau \, \text{sinc}(f\tau) \cdot e^{-J2\pi f \frac{\tau}{2}}$

$\quad = A\tau \, \text{sinc}(f\tau) \cdot e^{-J\pi f\tau}$

⑥

Find F.T. of $g(t)$ as shown below

$$g(t) = A \, \text{rect}\left(\frac{t - T/2}{\tau}\right) * - A \, \text{rect}\left(\frac{t + T/2}{\tau}\right)$$

using superposition & timeshift

$$A \, \text{rect}\left(\frac{t - T/2}{\tau}\right) \rightleftharpoons A\tau \, \text{sinc}(f\tau) \cdot e^{-j2\pi f T/2}$$

$$A \cdot \text{rect}\left(\frac{t + T/2}{\tau}\right) \rightleftharpoons A\tau \, \text{sinc}(f\tau) \cdot e^{+j2\pi f T/2}$$

$$G(f) = A\tau \, \text{sinc}(f\tau) \cdot \left[e^{-j\pi f \tau} - e^{+j\pi f \tau} \right]$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

⑦

⑤ Frequency - Shift Property

If $\quad g(t) \Longleftrightarrow G(f)$

then $\quad g(t) \cdot e^{\mp j2\pi f_0 t} \Longleftrightarrow G(f \mp f_0)$

<div dir="rtl">عكس الإشارة</div>

Find F.T. for $g(t) = A \, rect\left(\frac{t}{\tau}\right) \cdot e^{-j2\pi f_0 t}$

using frequency-shift

$\because A \, rect(t/\tau) \Longleftrightarrow A\tau \, sinc(f\tau)$

$\therefore G(f) = A\tau \, sinc\left((f+f_0)\tau\right)$

Find F.T. $\quad g(t) = A \, rect(t/\tau) \cdot cos(2\pi f_0 t)$.

$g(t) = \frac{1}{2}\left[A \, rect\left(\frac{t}{\tau}\right) \cdot \left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right)\right]$

$\qquad cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

$\quad = \frac{1}{2}\left[A \, rect\left(\frac{t}{\tau}\right) \cdot e^{j2\pi f_0 t}\right.$

$\qquad sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

$\qquad\left. + A \, rect\left(\frac{t}{\tau}\right) \cdot e^{-j2\pi f_0 t}\right]$

using freq. Shift & linearity

⑧

$$G(f) = \frac{1}{2}\left[A\tau\, \text{sinc}\left((f-f_0)\tau\right) + A\tau\, \text{sinc}\left((f+f_0)\tau\right) \right].$$

$$\underset{f=0}{\underbrace{\qquad}}$$

A rect$\left(\frac{t}{\tau}\right)$

$A\tau\, \text{sinc}\left(f\tau\right)$

$f = 0$
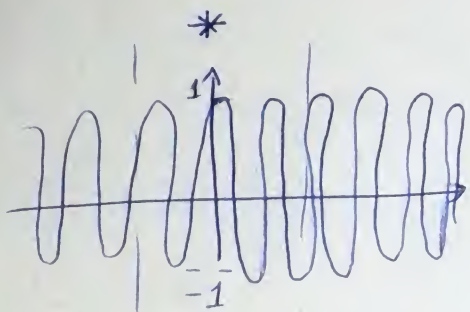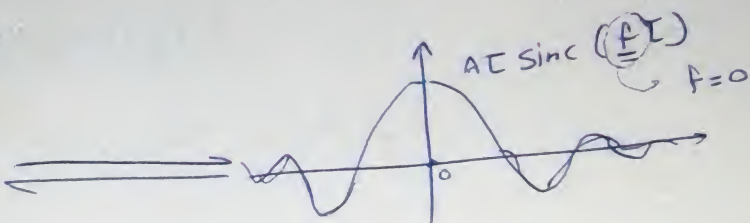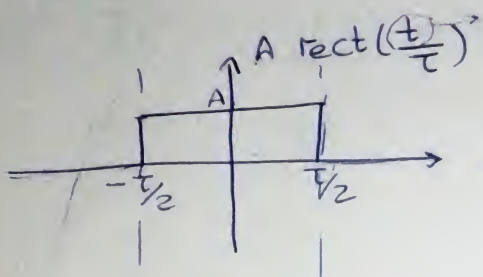
$\text{sinc}(0) = 1$

$f = 0 \rightarrow$ max. sine

$\text{sinc}(x) = \dfrac{\sin \pi x}{\pi x}$

$x = 0$

$\dfrac{\sin 0}{0} = 1$

$\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$

$g(t)$

$G(f)$

$A\tau/2$
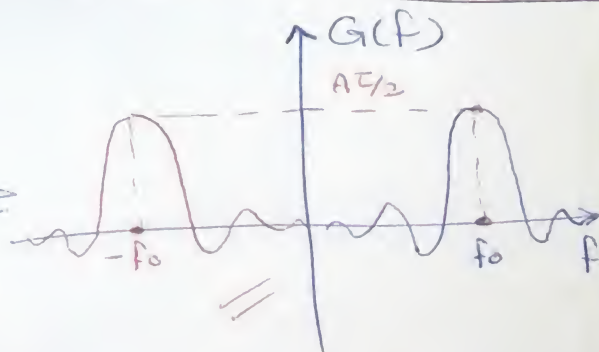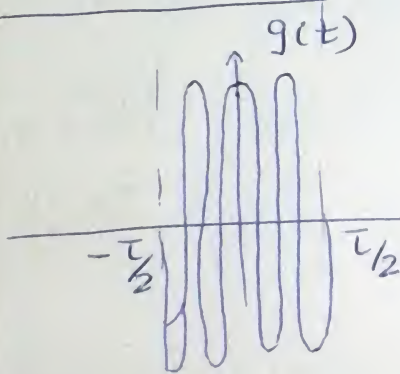
$$m(t)\cdot \cos(2\pi f_c t) \rightleftharpoons \frac{1}{2}\left[M(f-f_c) \oplus M(f+f_c) \right]$$

$$\text{sinc}$$

$$m(t)\cdot \sin(2\pi f_c t) \rightleftharpoons \frac{1}{2j}\left[m(t)\cdot e^{j\omega_c t} - m(t)\cdot e^{-j\omega_c t} \right]$$

$$\left(\xrightarrow{F.T.} \quad \frac{1}{2j}\left[M(f-f_c) \ominus M(f+f_c) \right] \right.$$

⑨

⑥ Area under curve g(t)

$$\text{Area} = \int_{-\infty}^{\infty} g(t) \cdot dt$$

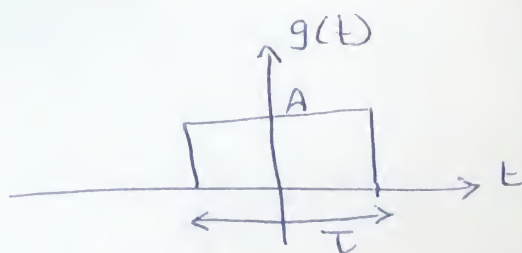$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-J2\pi f t} \cdot dt$$

at $f = 0$ $\boxed{\text{Area} = G(0)}$

Find the area under curve $g(t) = A \operatorname{rect}(t/\tau)$.

$\therefore G(f) = A\tau \operatorname{sinc}(f\tau)$.

$G(0) = A\tau \operatorname{sinc}(0)$
$\qquad\qquad\qquad \searrow 1$

$\therefore \text{Area} = G(0) = A\tau$.

Area = $A\tau$.

---

⑦ Area under the Curve $G(f)$

$$Area = \int_{-\infty}^{\infty} G(f) \cdot df$$

$$g(t) = \int_{-\infty}^{\infty} G(f) \cdot e^{+j2\pi f t} \cdot df$$

At $t = 0$

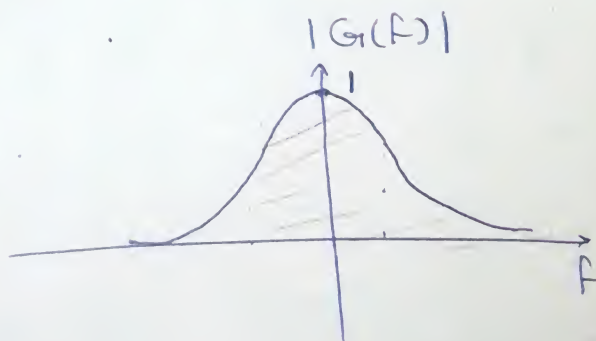$$\underline{Area = g(0)}$$

Find Area under $G(f) = \dfrac{1}{1 + j2\pi f}$

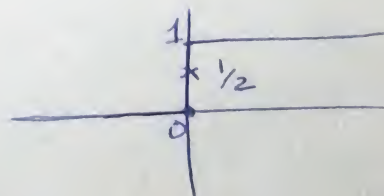$\bar{e}^{t} \cdot u(t) \rightleftharpoons \dfrac{1}{1 + j2\pi f}$

$$|G(f)| = \dfrac{1}{\sqrt{1 + 4\pi^2 f^2}}$$

$|G(f)|$

$\therefore g(t) = \bar{e}^{t} \cdot u(t)$

$\therefore$ Area under $G(f) = g(0)$

$\qquad\qquad\qquad = e^{0} \cdot u(0)$

$\qquad\qquad\qquad = 1 \cdot \dfrac{1}{2}$

$\therefore \boxed{Area = \dfrac{1}{2}}$

⑪